HIGHER HARMONIC RF SYSTEM FOR APS

The influence of a higher harmonic rf system was considered previously[1] but was treated as a non-accelerating system. This note treats the higher harmonic system more generally and treats the non-linear synchrotron oscillations without the use of a power series expansion.

The rf voltage seen by the beam is assumed to be the sum of one or two rf systems with a harmonic relationship to each other, given by the integer $n > 1$. The frequencies are then $f_2 = nf_1$. Although this need not be the case (i.e., f_1 and f_2 need only be harmonically related to f_o), operationally this harmonic relationship makes the rf systems easier to phase lock and assures that all buckets will have similar properties. The voltage provided by this rf system is

$$V(\phi) = V_1\{\sin(\phi + \phi_1) + k\sin(n\phi + n\phi_2)\} \qquad (1)$$

where

ϕ_1 = the synchronous phase of the main rf system

ϕ_2 = the synchronous phase of the n-th harmonic rf system

V_1 = the peak voltage of the main rf system

kV_1 = the peak voltage of the n-th harmonic rf system.

The longitudinal equation of motion[2] is given by

$$\ddot{\phi} + \frac{\Omega_o^2}{V_1\cos\phi_1}\{V(\phi) - U_o\} = 0, \qquad (2)$$

where U_o = the average synchrotron radiation loss per turn and

$$\Omega_o^2 = \omega_o^2 \left[\frac{-h_1 \eta V_1 \cos\phi_1}{2\pi E_o} \right]$$

is the small amplitude synchrotron frequency for the main rf system alone, with synchronous phase ϕ_1.

Equation (2) can be integrated over ϕ to give the Hamiltonian

$$(\frac{\dot{\phi}}{\Omega_o})^2 + \frac{2Y^2(\phi,\phi_1,\phi_2)}{\cos\phi_1} = H \qquad (3)$$

with

$$Y^2(\phi,\phi_1) = \frac{1}{V_1} \int_o^\phi \{V(\phi) - V_r\} d\phi$$

The bunch current will be given by[3]

$$i(\phi) = A \exp\left[\frac{-eV_1 Y^2(\phi,\phi_1,\phi_2)}{2\pi h_1 \eta E_o (\frac{\sigma_E}{E})^2} \right] . \qquad (4)$$

The bunch length will be a maximum and have a symmetric distribution if the first and second derivative of $V(\phi)$ at $\phi = 0$ are zero. This yields the constraints[2]

$$nk \cos n\phi_2 = -\cos\phi_1. \qquad (5)$$

and

$$n^2 k \sin n\phi_2 = -\sin\phi_1. \qquad (6)$$

In addition, the energy constraint requires $V(\phi=0) = U_o$ or

$$k \sin n\phi_2 = \frac{U_o}{V_1} - \sin\phi . \qquad (7)$$

If insufficient voltage is available from the higher harmonic system (to solve both Eqs. (5) and (6)), then Eq. (6) alone can be solved to provide longer symmetric bunches than would be possible from the main rf system. For this condition, Eqs. (6) and (7) become

$$\sin\phi_1 = \frac{U_0}{V_1} \left(\frac{n^2}{n^2-1}\right) \tag{8}$$

and

$$\sin n\phi_2 = \frac{-U_0}{kV_1(n^2-1)} . \tag{9}$$

Note that these conditions require a minimum voltage from the higher harmonic system of $kV_1 > U_0/(n^2-1)$ and $V_1 > U_0 n^2/(n^2-1)$. If the voltage available from the higher harmonic system is large enough, the maximum bunch lengthening can be achieved by satisfying the constraints given by Eqs. (5) and (6) to yield the constraints

$$\tan n\phi_2 = \frac{1}{n} \tan \phi_1 \tag{10}$$

$$k = \frac{-\cos \phi_1}{n \cos n\phi_2} \tag{11}$$

and Eq. (8). The only free parameter, V_1, is then adjusted to yield sufficient bucket height for quantum and Touschek lifetime. Since the solution to Eq. (3) is not easily calculated analytically, the solution will be performed numerically to determine the synchrotron frequency, f_s, by

$$\frac{1}{f_s} = 2 \int_{\phi_a}^{\phi_b} \frac{d\phi}{\dot{\phi}} ,$$

where ϕ_a and ϕ_b are the extreme values of ϕ for a momentum error $\Delta E/E$, obtained by solving Eq. (3) for $\dot{\phi}_{max} = -2\pi f_0 h_1 \eta \frac{\Delta E}{E}$ and $\dot{\phi} = 0$.

Bunch Lengthening for APS

Assuming the beam and the main rf parameters given in CDR-'87: $E_0 = 7$ GeV, $U_0 = 6.9$ MeV/turn, $h_1 = 1248$, $V_1 = 9.5$ MV, and $n = 3$ (i.e., $h_2 = 3h_1$), then the maximum bunch lengthening given by solving Eqs. (7), (10), and (11) for $V_2 = 2.02$ MV ($k = 0.2125$), $\phi_1 = 125.2°$ and $\phi_2 = -8.43°$.

Note that ϕ_2 is defined as a phase angle for the first harmonic, thus, the power given to/from the beam is

$$P_2 = I_0 V_2 \sin(3\phi_2) \tag{12}$$

which is negative (i.e., energy absorbed from the beam in the rf cavity and reflected back toward the rf generator).

Figure 1(a and b) shows the potential function $\frac{2Y^2(\phi, \phi_1)}{\cos \phi_1}$ and the voltage for the maximum bunch lengthening condition. Figure 2(a and b) shows the current profile calculated from Eq. (4), for this maximum bunch lengthening condition and for the main rf system alone ($V_1 = 9.5$ MV, $\phi_1 = 133.4°$). Since the current profile for the bunch with two-rf systems isn't Gaussian, the bunch length is determined by taking the same fractional current height, $e^{-0.5}$. The zero current bunch length for a single rf system is $\sigma_\ell \approx 0.6$ cm, while the maximum length is $(\sigma_\ell)_{max} = 3.3$ cm. Figure 3 shows the rf bucket (separatrix) and the $\left(\frac{\Delta E}{E}, \phi\right)$ trajectory for the natural energy spread $\sigma E/E = 0.096\%$. Figure 4 shows the calculated synchrotron frequency for this double rf system.

In the CDR-'87, V_2 = 1.8 MV limits the bunch lengthening to that shown in Figs. 5 and 6. If the synchronous phase of the second rf system is maintained at zero (no power transfer from the beam) the bunch length is shown in Fig. 7 and is more asymmetric at the larger phase shifts than the case in Fig. 5. The synchrotron frequency for this case is shown in Fig. 8. Table I summarizes the effect of the third harmonic system on bunch length and f_s. Notice that for a fixed V_1, the bucket height is actually reduced; consequently, V_1 should be increased to maintain a constant beam lifetime. However, the $\Delta E/E$ bucket height of 2% was required to provide for a factor of three increase in bunch length due to turbulent bunch lengthening. The second rf system provides the bunch lengthening required without increasing the energy spread as much as this factor of 3, consequently the 1.8% bucket height should be more than adequate for sufficient lifetime.

Table I

RF Systems for U_o = 6.9 MeV/Turn with V_1 = 9.5 MV

RF System	n	ϕ_1	ϕ_2	V_2 (MV)	σ_ℓ^* (cm)	f_s^* (kHz)	$\left(\Delta E/E\right)_{max}$	$V_2\sin(n\phi_2)$ (MV)
Single	–	133.4°	–	–	0.58	1.88	2.0%	–
Double	3	125.2°	−8.4°	2.02	3.3	0.403	1.79%	−0.851
Double	3	125.2°	−9.5°	1.8	1.63	0.670	1.82%	−0.859
Double	3	133.4°	0	1.8	1.34	0.781	1.8%	0.0

*For $\Delta E/E$ = 0.096%, the natural energy spread.

Effect on Single Bunch Current

The single-bunch current limits for the fast blowup instabilities are given by[4]

$$I_p^{\parallel} \; < \; \frac{4\pi E \; \alpha}{e\left|\frac{Z_{\parallel}}{n}\right|} \left(\frac{\sigma_E}{E}\right)^2 \qquad \text{longitudinal} \qquad (13)$$

and

$$I_p^{\perp} \; < \; \frac{4\pi\sqrt{2\pi} \; E \; \alpha}{e\left|\frac{Z_{\perp}}{n}\right|\bar{\beta}} \left(\frac{\sigma_E}{E}\right) \qquad \text{transverse} \qquad (14)$$

where $I_p^{\parallel}$ and $I_p^{\perp}$ are the peak current in the bunch, and the impedances are expressed as peak impedance of a broadband resonator with cutoff frequency $w_c \approx \frac{c}{2r} = 2\pi f_o$ n. In these expressions, the peak current is independent of the bunch length and only depends on the natural energy spread and the impedance at the cutoff frequency. Then bunch lengthening from the high harmonic system yields an increase in the average current per bunch given by the ratio of integrals of the two bunch profiles shown in Fig. 2. Figure 9 shows the dependence of the gain in the average bunch current for a double rf system relative to that from a single rf system with the same peak current, as a function of the voltage from the third harmonic system. The maximum gain in the average current is obtained for V_2 = 2.0 MV and is 4.8 times that obtained with the single rf system.

The transverse coupled-mode instability has been observed as limiting the single-bunch current in several high-energy-storage electron rings. The increase in bunch length by a higher harmonic system has been shown to yield a factor of five increase in this current limit in PETRA.[5] Since most analyses of the coupled-mode instability assume Gaussian distributions, the gain in this threshold current will be treated in a future note.

Bunch Shortening for the APS

In addition to lengthening of the bunch, the higher harmonic system can also be used to shorten the bunch for experiments that require shorter pulses of synchrotron lights. The maximum bunch shortening is obtained by setting the second derivative of $V(\phi)$ equal to zero at $\phi=0$. This is the same as Eq. (9) but with the solution

$$n\phi_2 = \pi - \sin^{-1}[\frac{-U_o}{V_1 k(n^2-1)}]$$

which maximizes dV/dt at $\phi=0$. In this case, the voltage can be approximated by a linear dependence in ϕ and the synchrotron frequency approximated by the small amplitude frequency given by

$$f_s = f_o[\frac{\alpha e}{2\pi E_o} h_1 V_1(\cos \phi_1 + nk \cos n\phi_2)]^{1/2}.$$

Then the bunch length (rms pulse duration) is given by

$$\sigma_\tau = \frac{\alpha}{2\pi f_s} (\frac{\sigma E}{E})$$

where $\sigma E/E$ is the natural energy spread. Figure 10 shows the dependence of σ_τ on the rf voltage from a third harmonic system.

In this calculation, $V_1 = 9.5$ MV was assumed and therefore the bucket height increased as V_2 was raised. In the bunch shortening mode, power must be supplied by the third harmonic system. For the case with $V_2 = 1.8$ MV, $\sigma_\tau = 15.5$ ps, and the power given to the beam is 8.6 kW for a 10-mA beam current. The average current per bunch in this mode of very short bunches will be quite small due to transverse instabilities and also to prevent longitudinal turbulence from lengthening the bunch. The total current will then depend on the users requirement for the dark time between light pulses.

Conclusion

The addition of a higher harmonic rf system provides an independent and efficient means of varying the bunch length either longer or shorter than that possible from the main rf system. The main rf system provides most of the power to the beam to counter the synchrotron radiation losses. In the bunch lengthening mode, the only power required from the higher harmonic system is to determine the phase of the rf voltage, but power is actually absorbed from the beam toward the generator. Consequently, a circulator and high-power load are required for this system. In the bunch shortening mode, power must be provided by the generator to the beam. This power may be small, since the average currents in this mode will probably be significantly smaller than in the single rf system case.

The system specified in CDR'87 can provide for a factor of 2.3 increase in the rms bunch length or a 20% reduction in the rms bunch length. In the bunch lengthening, the increase in single bunch current will be almost a factor of 3 for the turbulent instability limit. The increase in Landau damping due to synchrotron tune spread will provide an increase in bunch and total current for the coupled-mode instabilities.

Phase stability of the third harmonic system relative to the main rf will be critical, especially during injection. This has been handled at PETRA by developing a feedback system to detect the bunch asymmetry and correcting ϕ_2 by this signal.[5] In the bunch shortening mode, this phase stability will be more critical and may require a different detector system, for example, to detect high frequency bunch motion in a dispersive section.

References

1. T. K. Khoe, "The Effects of Adding a Higher Harmonic Cavity," ANL Report LS-63 (1986).

2. A. Hofmann and S. Myers "Beam Dynamics in a Double RF System," 11th Int. Conf. on High Energy Accelerators, CERN, p. 610 (1980).

3. J. Haissinski, Il Nuovo Cimento, **18B** (1973).
   A. Papiernik, et al., Ninth International Conf. on High Energy Accelerators, SLAC, p. 375 (1974).

4. Ron Ruth, "Single Bunch Instabilities in an SSC," Accelerator Physics Issues for a Superconducting Super Collider, UMHE84-1, Ann Arbor (1983).

5. R. D. Kohaupt, "The Experimental Study of a High Harmonic RF System in PETRA," IEEE NS-30, 2525 (1983).

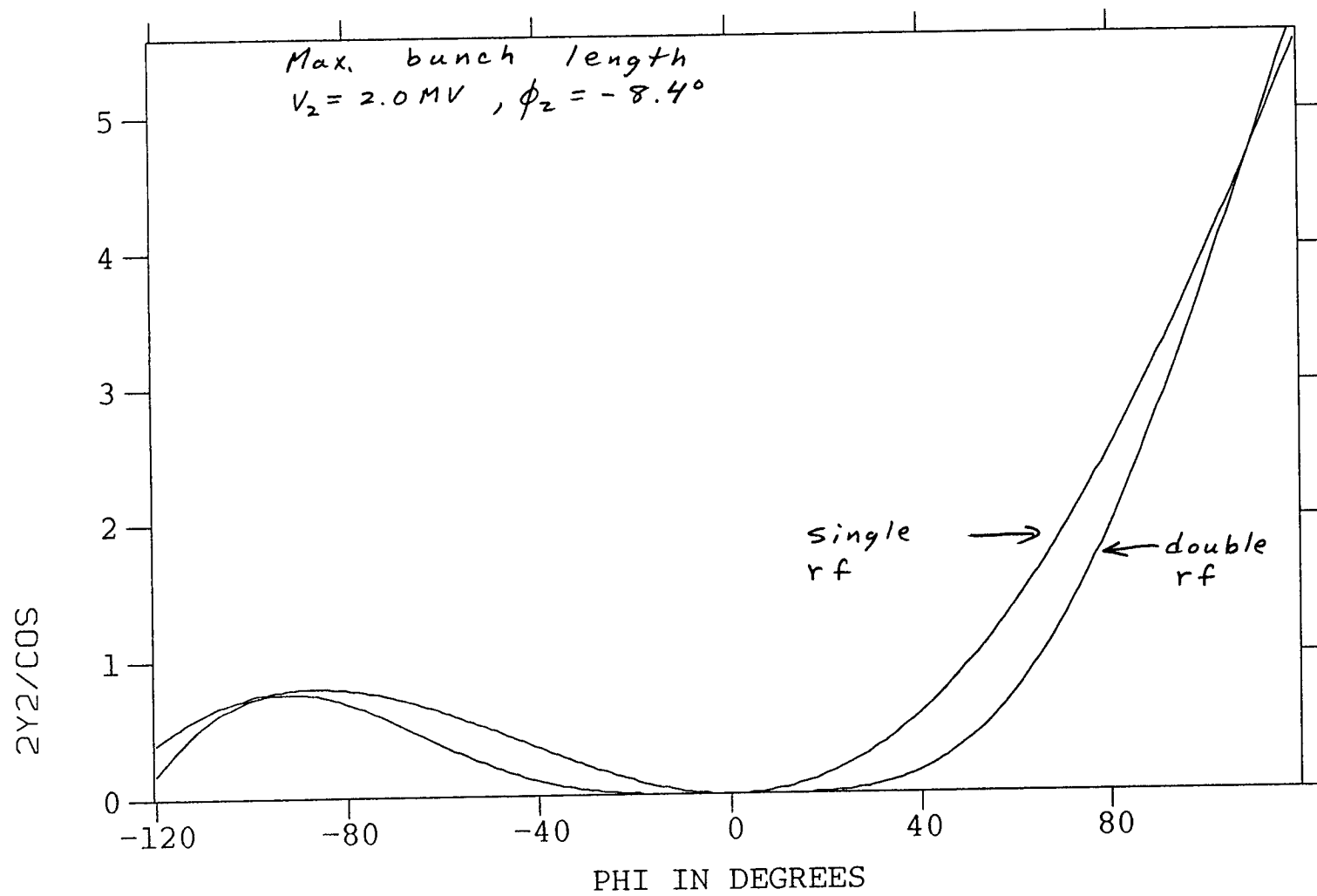Figure 1(a)   POTENTIAL FUNCTION

Max. bunch length
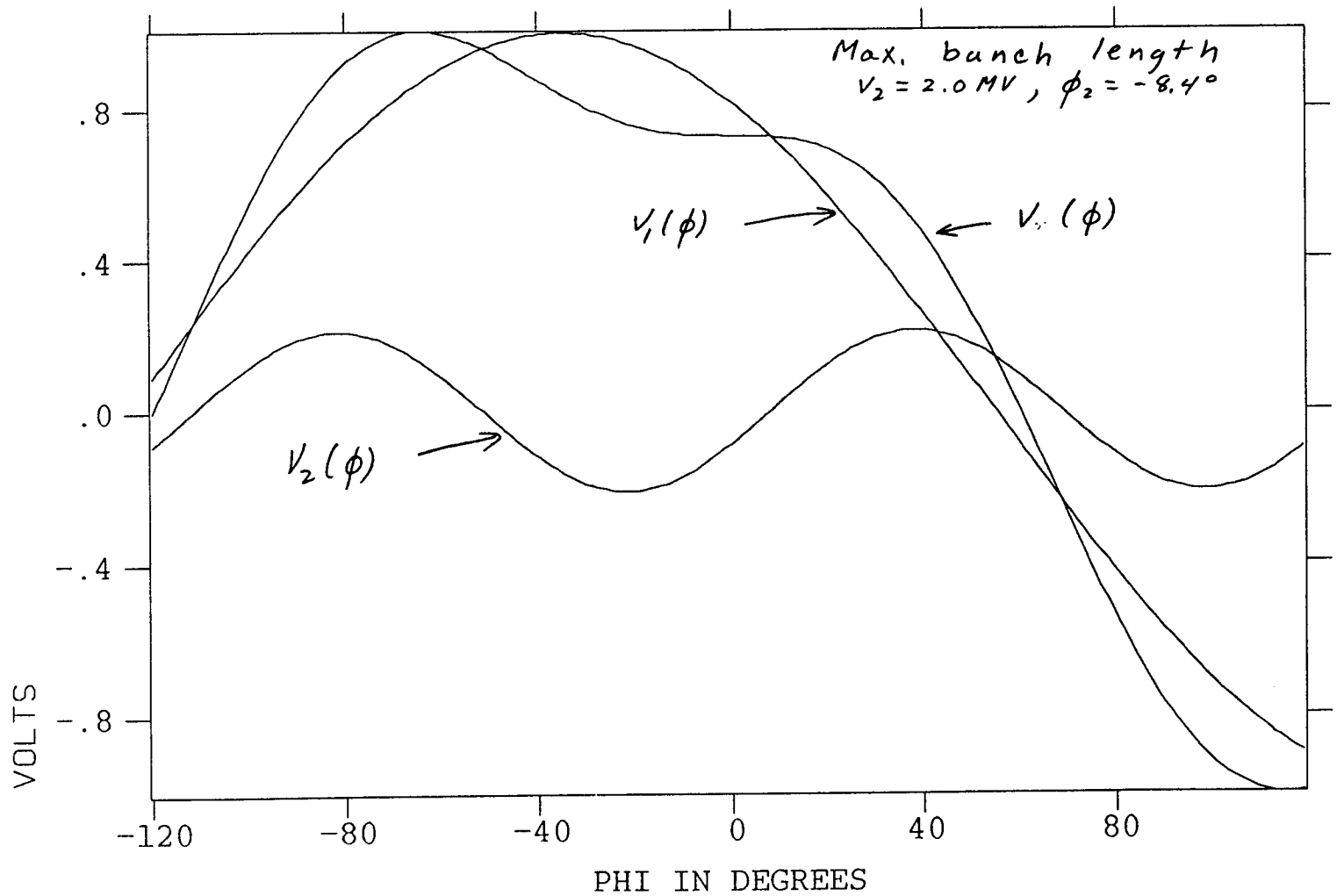$V_2 = 2.0\,MV$, $\phi_2 = -8.4°$

single rf

double rf

2Y2/COS

PHI IN DEGREES

HANDYPAK   09:42:33   18MAY87

Figure 1(b)   RF-VOLTAGE PLOTS

Max. bunch length
$V_2 = 2.0 \, MV$, $\phi_2 = -8.4°$

$V_1(\phi)$

$V_2(\phi)$

$V_2(\phi)$

VOLTS

PHI IN DEGREES

HANDYPAK   09:42:33   18MAY87

Figure 2(a)   BEAM LENGTH PROFILE

Max. bunch length
$V_2 = 2.0 MV$, $\phi_2 = -8.4°$

$\sigma_\ell)_0 = 0.6\ cm$

$\sigma_\ell)_{max} = 3.3\ cm$

CURRENT

POSITION IN MM.

HANDYPAK   09:42:33   18MAY87
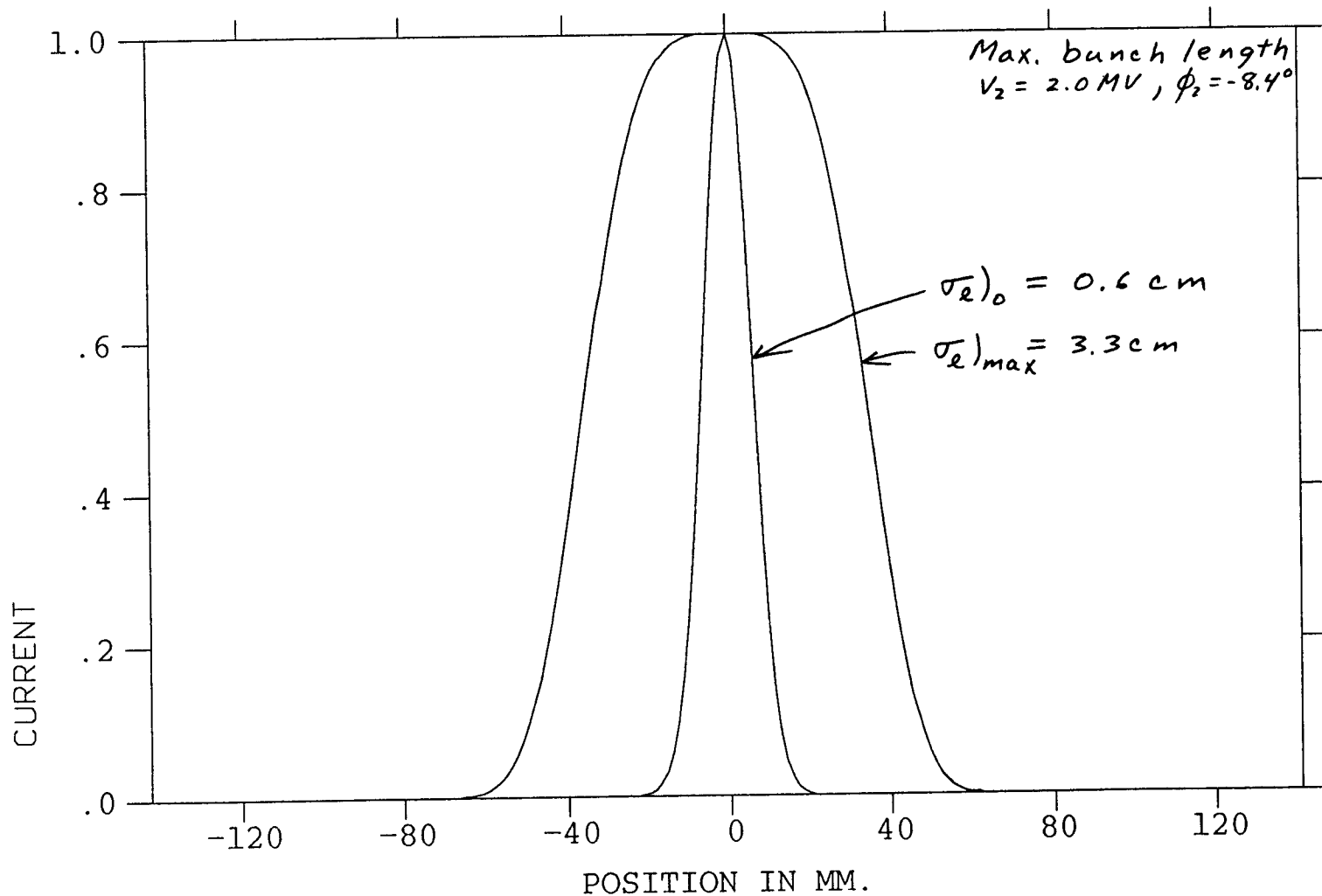
Figure 2(b)

BEAM TIME PROFILE

Max. bunch length

$\sigma_\ell)_0 = 0.6$ cm

$\sigma_\ell)_{max} = 3.3$ cm

CURRENT

TIME IN NSEC.

HANDYPAK   09:42:33   18MAY87
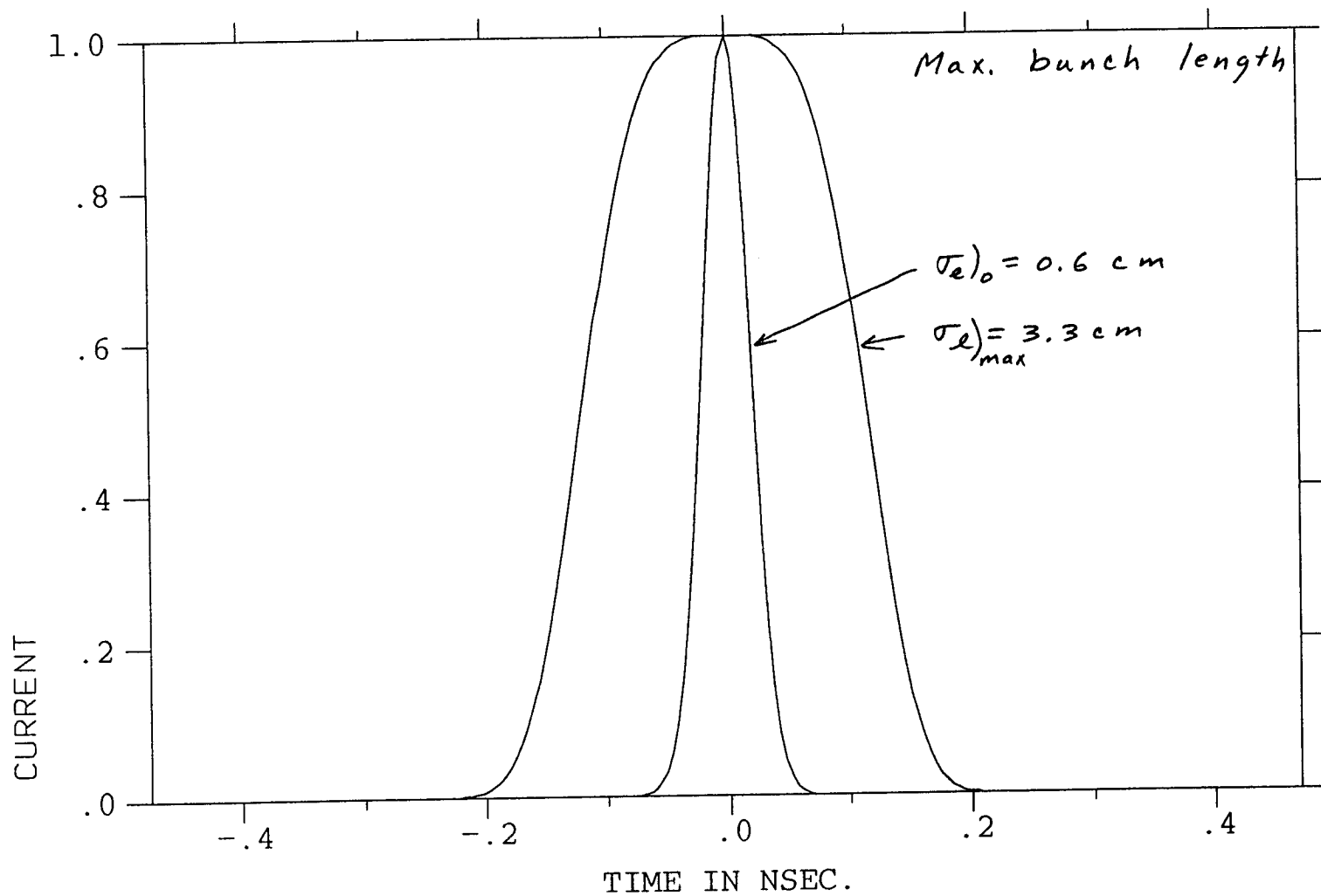
Figure 3   Synch phase space

$V_2 = 2.0\,MV,\ \phi_2 = -8.4°$

$\dfrac{\Delta E}{E} = 0.096\%$

$\dfrac{\Delta E}{E} = 1.79\%$

DE/E (%)

Phi in Degrees

HANDYPAK   09:42:33   18MAY87
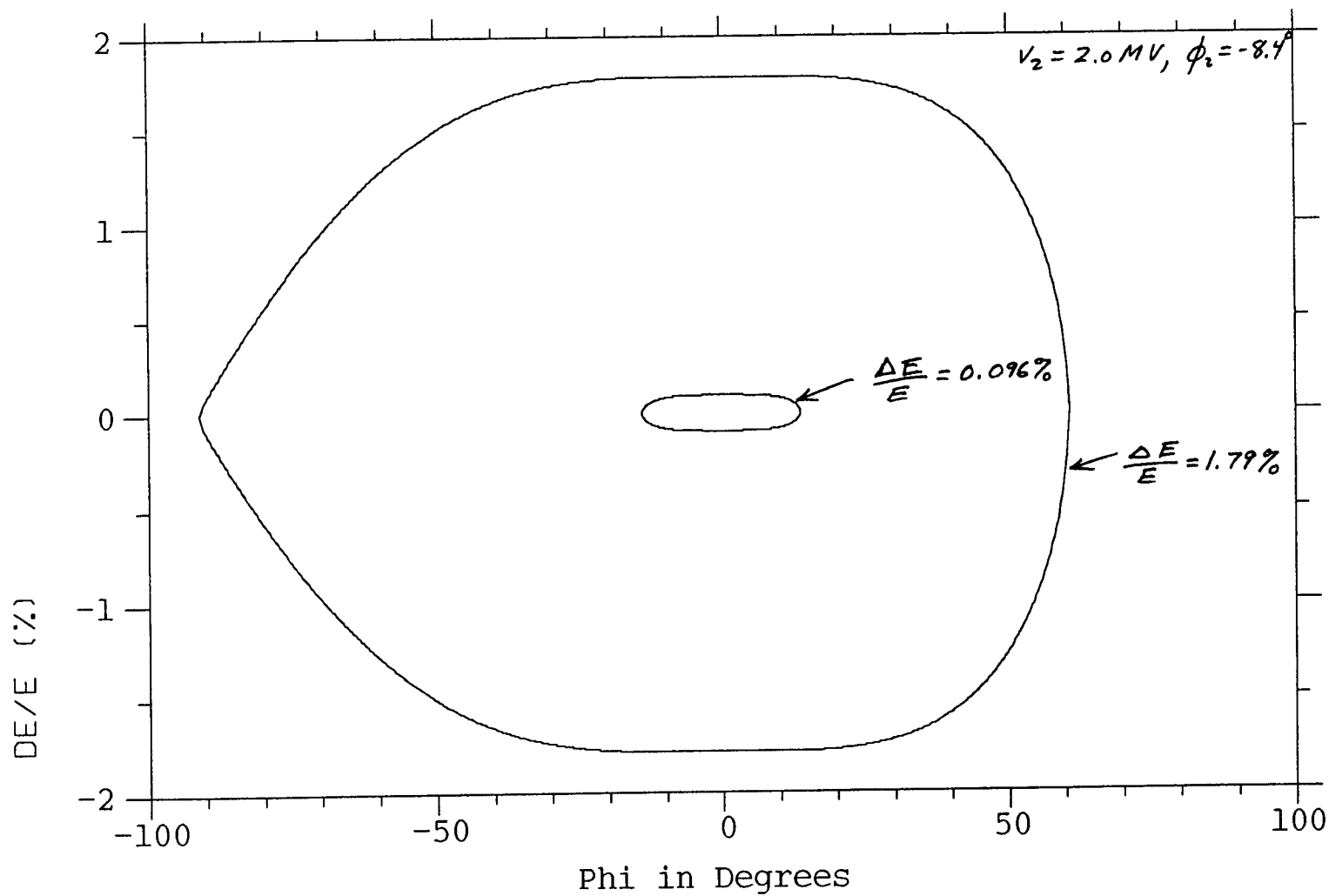
FIG. 4 Synch. Freq. for Max. Bunch Length

$V_2 = 2.0MV$, $\phi_2 = -8.4°$
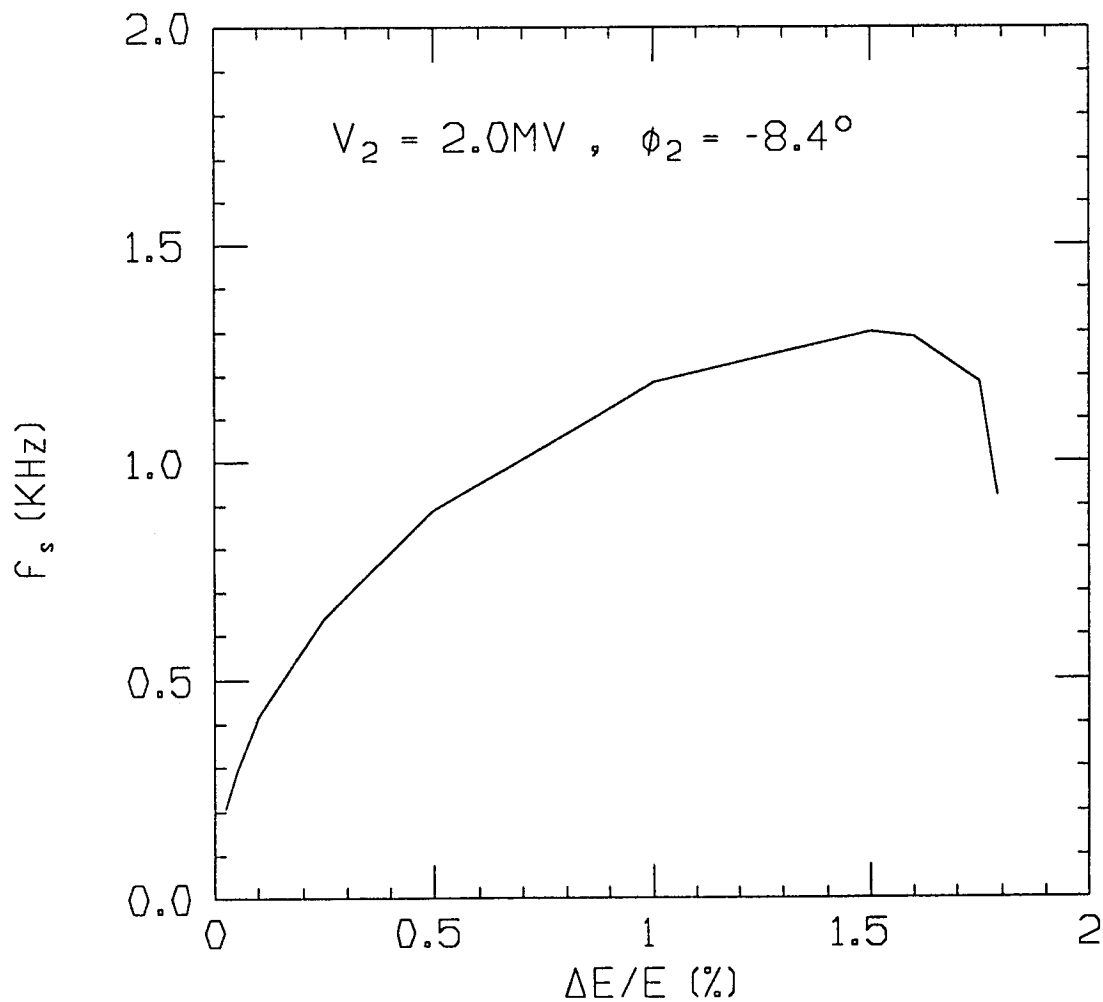
Figure 5

BEAM LEN. PROFILE V2=1.8

$V_2 = 1.8 MV, \quad \phi_2 = -9.5°$

$\sigma_\ell)_0 = 0.6 cm$

$\sigma_\ell)_2 = 1.62 cm$

CURRENT

POSITION IN MM.
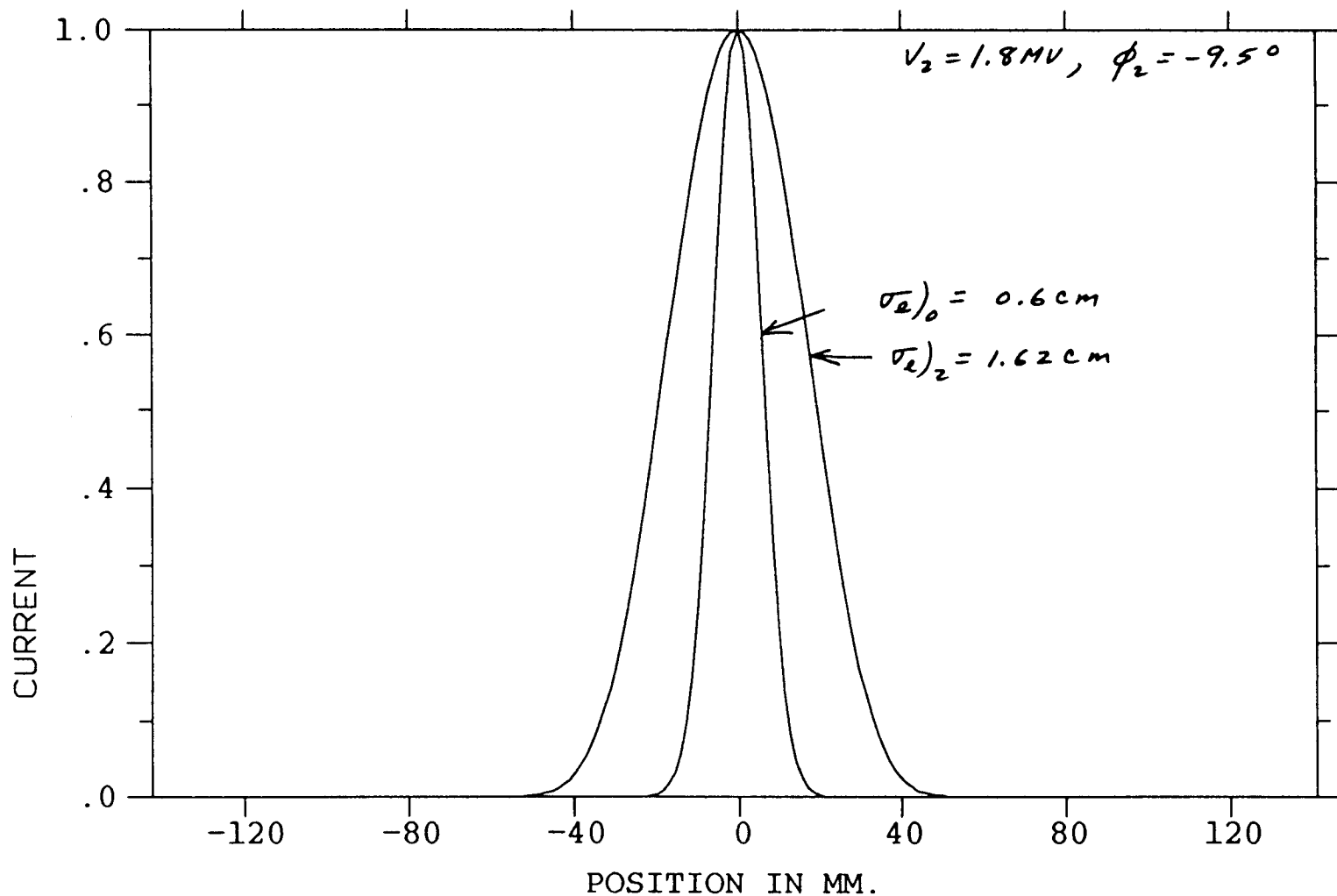
FIG. 6 Synch. Freq. for CDR-'87

$V_2 = 1.8MV$, $\phi_2 = -9.54°$
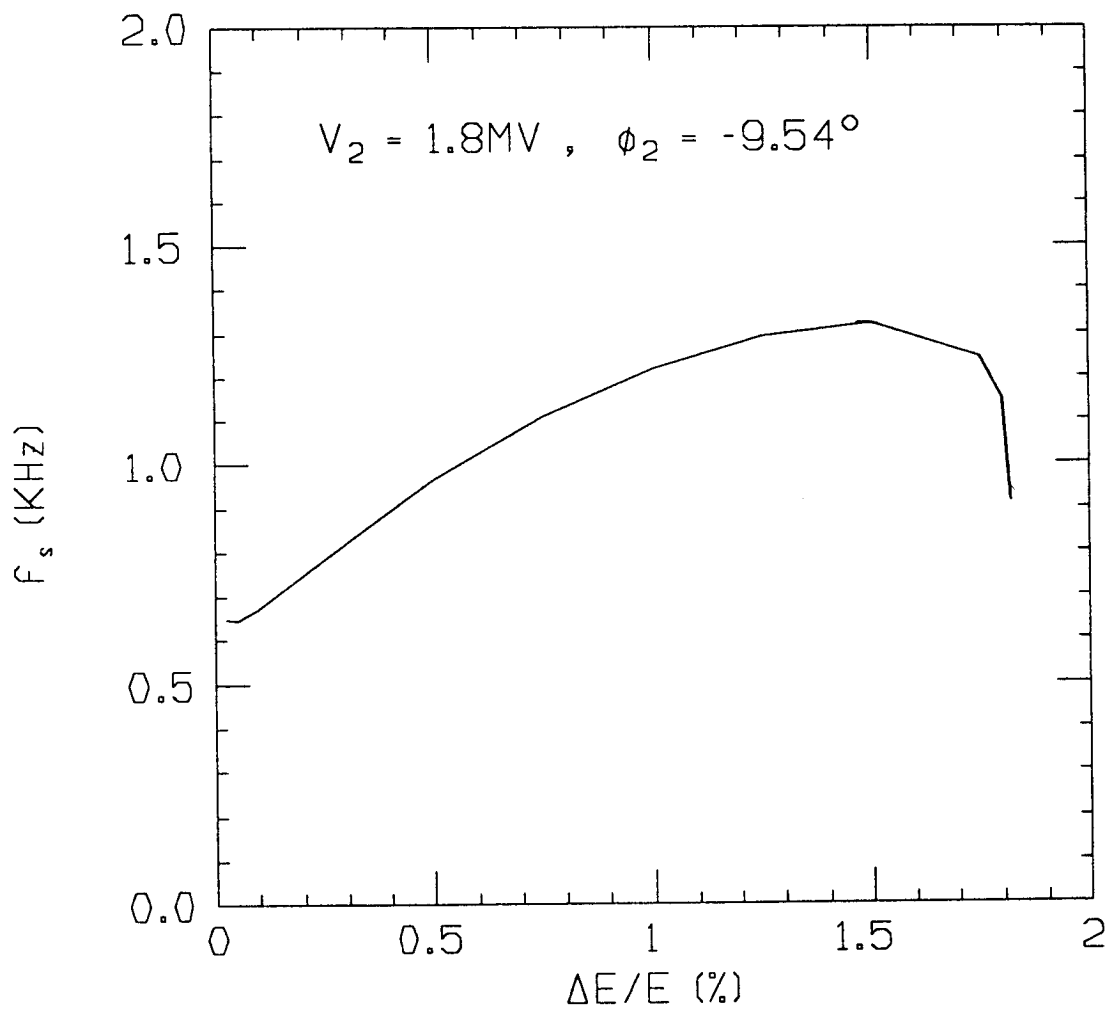
Figure 7   BEAM LIN. PROFILE V2=1.8

$V_2 = 1.8\, MV$, $\phi_2 = 0.0°$

$\sigma_\ell)_0 = 0.6\ cm$

$\sigma_\ell)_2 = 1.34\ cm$

CURRENT

POSITION IN MM.

HANDYPAK   14:02:44   18MAY87
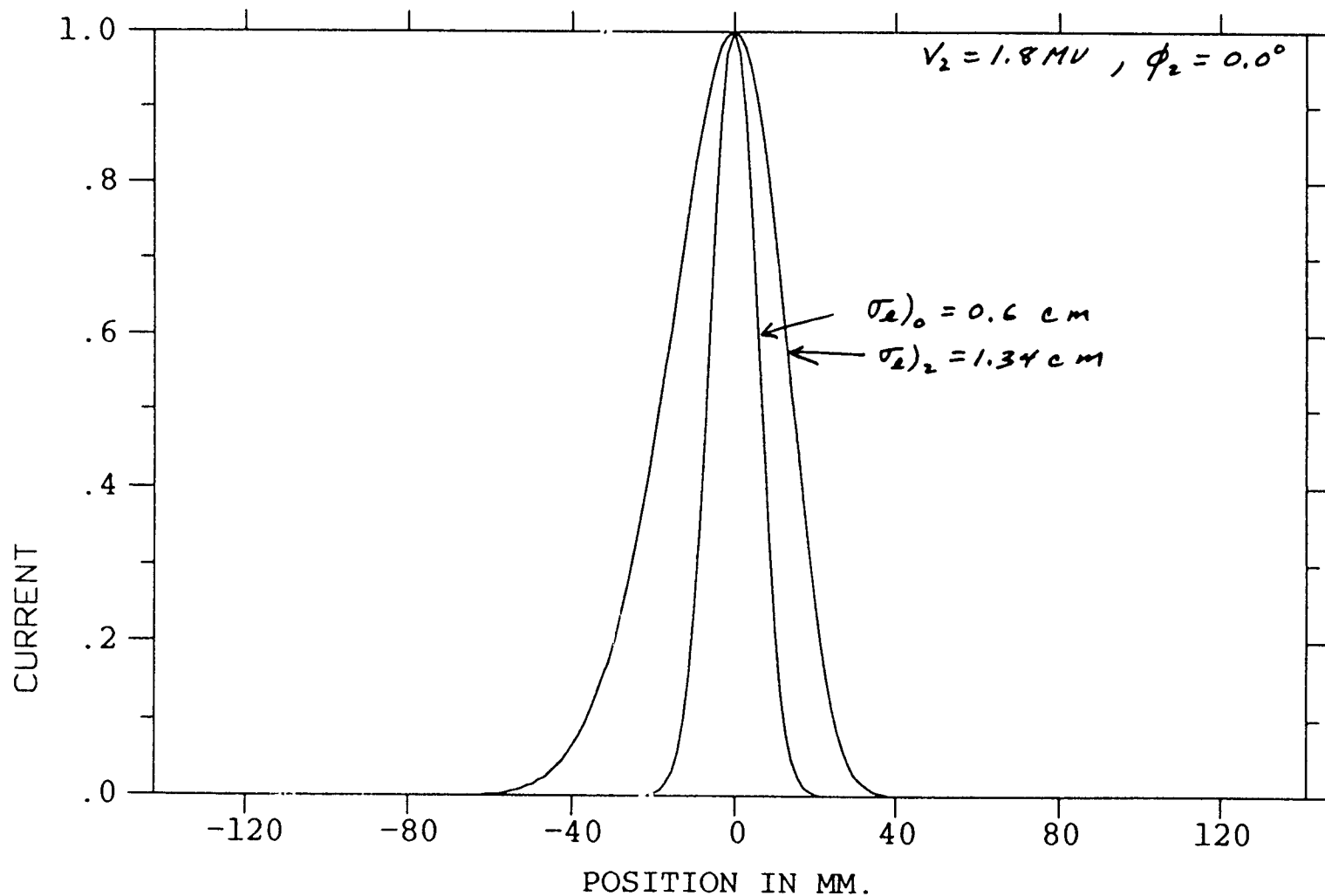
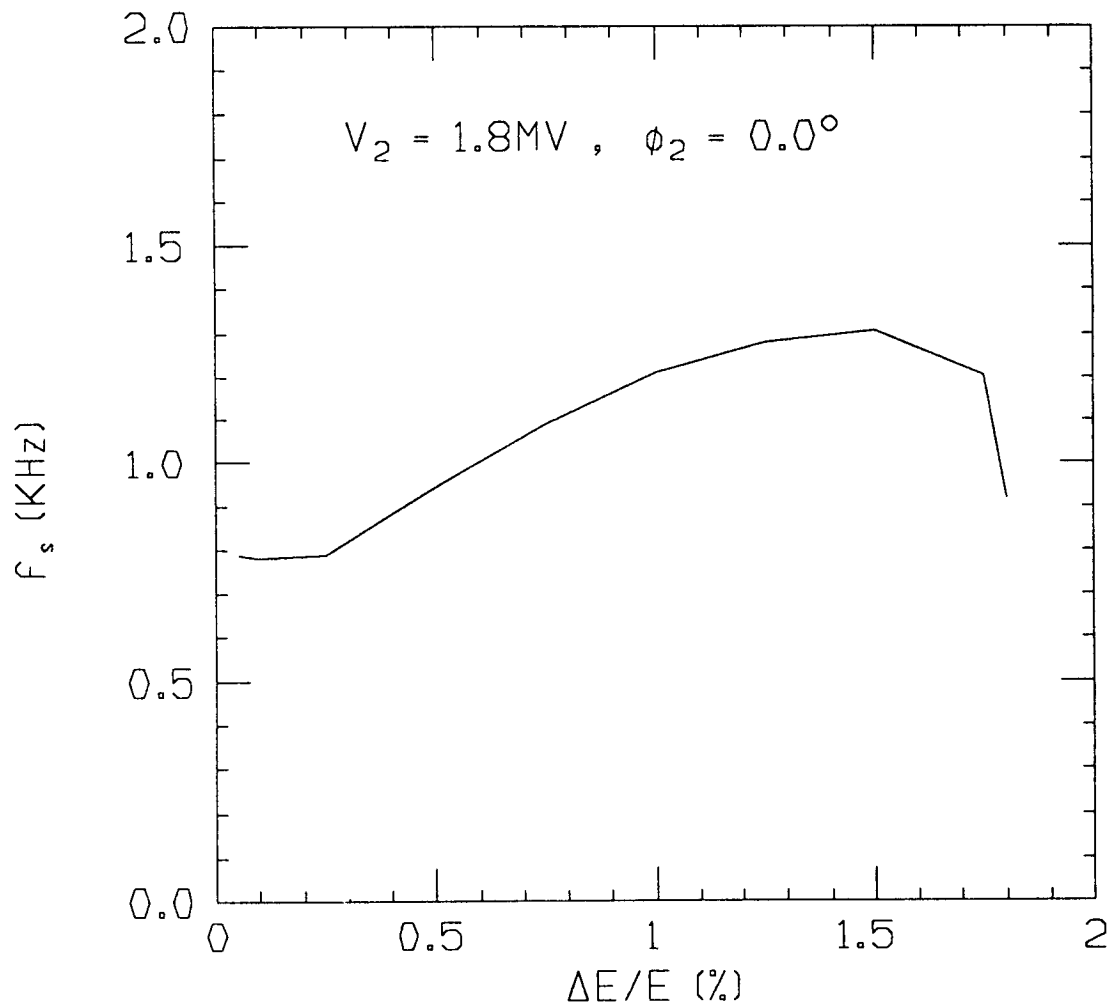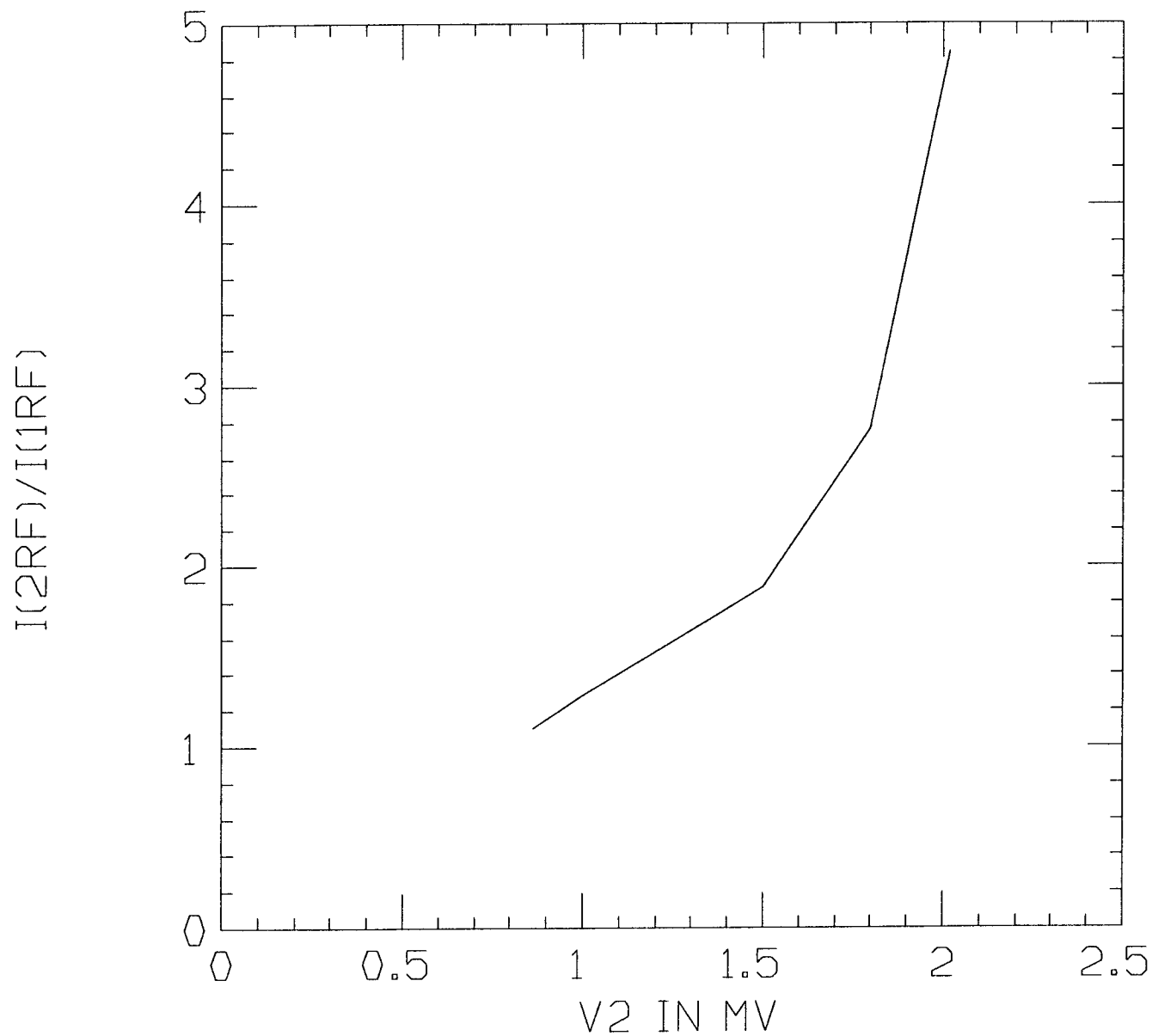FIG. 8 Synch. Freq. for CDR-'87

$V_2 = 1.8\text{MV}$, $\phi_2 = 0.0°$

FIG. 9 INCREASE IN AVERAGE CURRENT

FIG. 10 BUNCH LENGTH vs.V2